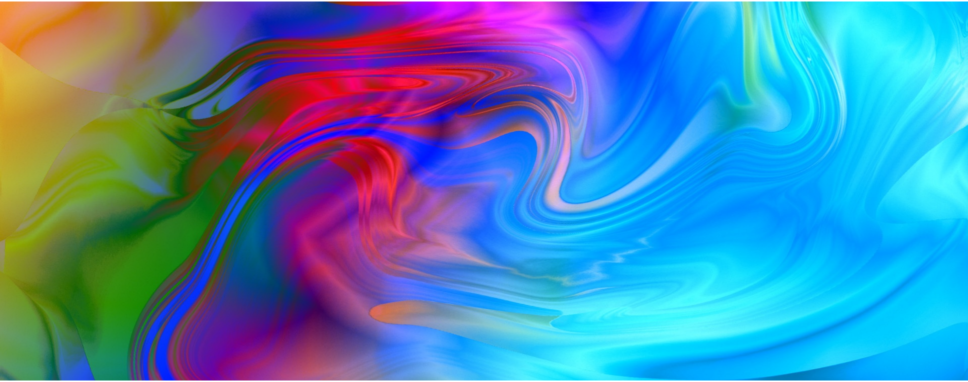

Multi-source Fusion Operations in Subjective Logic

FUSION Conference 2018

Rens W. van der Heijden,
Henning Kopp, Frank Kargl

2018-07-13

Motivation

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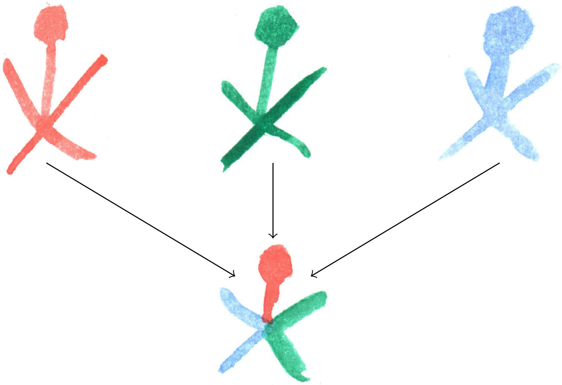

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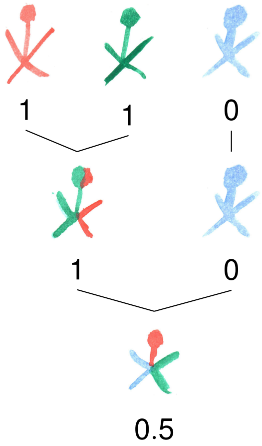

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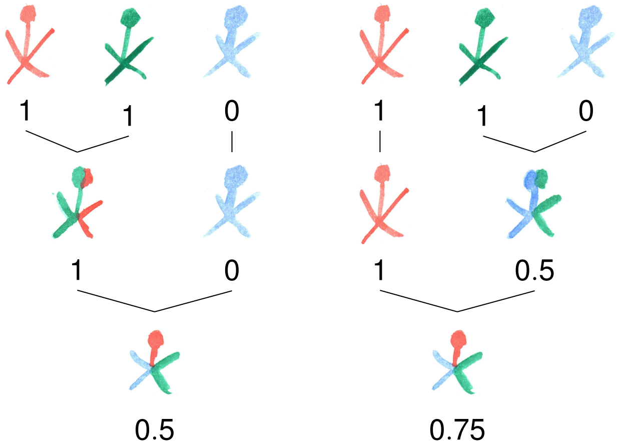

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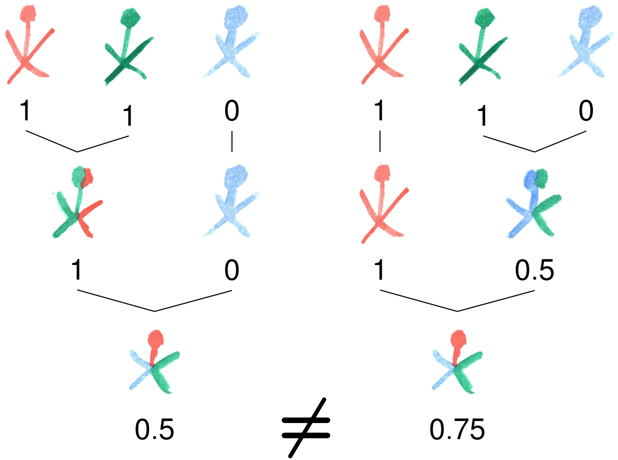

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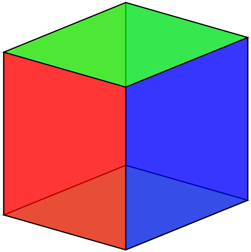
Subjective Logic Multinomial Example



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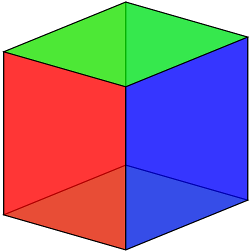

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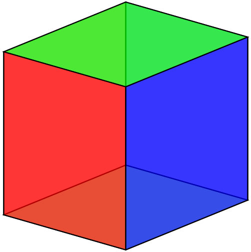

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Fusion?

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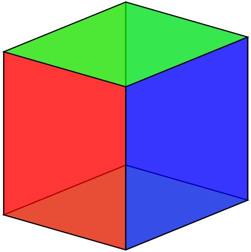
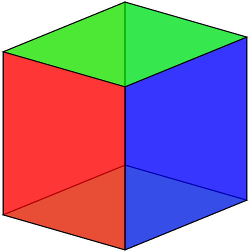


Domain $\mathbb{X} = \{\mathrm{r}, \mathrm{g}, \mathrm{b}\}$
Use ω_X^A to model whether the die is weighted, e.g., by repeated experiments
Fusion? $\rightarrow$ combine results from different actors

Subjective Logic Hyper-opinion Example



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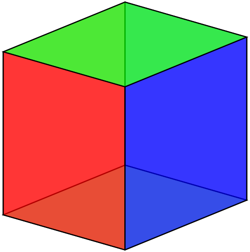

Domain $\mathbb{X} = \{\mathrm{r}, \mathrm{g}, \mathrm{b}\}$ for observations under:

- low light condition (green/blue indist.): $b_X^{LL}(\{\mathrm{g}, \mathrm{b}\})$
- colorblindness condition (red/green indist.): $b_X^{CB}(\{\mathrm{r}, \mathrm{g}\})$

$\rightarrow$ no assignment of relative belief (as in multinomial case)!

Subjective Logic Fusion

Available binary operators:

- Cumulative Belief Fusion
- Belief Constraint Fusion (aka Dempster's Rule)
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Multi-source generalizations do not exist for all these operators!

Subjective Logic Fusion: Previous Work

Jøsang et al. (FUSION 2017) provide:

- Cumulative Belief Fusion
- Belief Constraint Fusion (aka Dempster's Rule)
- Averaging Belief Fusion
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Subjective Logic Fusion: Commutativity

Dempster's Rule is commutative:

- Cumulative Belief Fusion
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Subjective Logic Fusion: Our Work

We define multi-source fusion operations for:

- Cumulative Belief Fusion
- Belief Constraint Fusion (aka Dempster's Rule)
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Subjective Logic Fusion: WBF generalization

Two-source:

$$\mathbf{b}_X^{\widehat{\diamond}\mathbb{A}}(x) = \frac{\mathbf{b}_X^A(x) \cdot c_X^A \cdot u_X^B + \mathbf{b}_X^B(x) \cdot c_X^B \cdot u_X^A}{u_X^A + u_X^B - \mathbf{2} u_X^A u_X^B}$$

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Multi-source:

$$\mathbf{b}_X^{\widehat{\diamond}\mathbb{A}}(x) = \frac{\sum_{A \in \mathbb{A}} \mathbf{b}_X^A(x) \left(1 - u_X^A\right) \prod_{A' \in \mathbb{A}, A' \neq A} u_X^{A'}}{\left(\sum_{A \in \mathbb{A}} \prod_{A' \neq A} u_X^A\right) - |\mathbb{A}| \cdot \prod_{A \in \mathbb{A}} u_X^A}$$

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Next steps:

- Efficient representations
- Computational efficiency of CCF

Questions?

Contact information

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Acknowledgments & Licenses

We thank the following people for the fruitful discussions that led to this paper:

- David Köhler (Ulm University)
- Audun Jøsang (University of Oslo)

Slide & TikZ templates are available here:
`https://github.com/vs-uulm/latexslides-uulm`
`https://github.com/vs-uulm/tikz-vanet`

Backup Slides

Multi-source Weighted Belief Fusion

Definition (WBF of opinions for multiple sources)

Let $\mathbb{A}$ be a finite set of actors and let $\omega_X^A = (\mathbf{b}_X^A, u_X^A, \mathbf{a}_X^A)$ denote the multinomial opinion held by $A \in \mathbb{A}$ over X. Then we define the weighted belief fusion of these opinions as the opinion $\omega_X^{\widehat{\diamond}\mathbb{A}} = (\mathbf{b}_X^{\widehat{\diamond}\mathbb{A}}, u_X^{\widehat{\diamond}\mathbb{A}}, \mathbf{a}_X^{\widehat{\diamond}\mathbb{A}})$ as follows:

Case 1: $(\forall A \in \mathbb{A} : u_X^A \neq 0) \land (\exists A \in \mathbb{A} : u_X^A \neq 1)$

$$\mathbf{b}_X^{\widehat{\diamond}\mathbb{A}}(x) = \frac{\sum\limits_{A\in\mathbb{A}} \mathbf{b}_X^A(x)\left(1 - u_X^A\right) \prod\limits_{A'\in\mathbb{A}, A'\neq A} u_X^{A'}}{\left(\sum\limits_{A\in\mathbb{A}} \prod\limits_{A'\neq A} u_X^A\right) - |\mathbb{A}| \cdot \prod\limits_{A\in\mathbb{A}} u_X^A}$$

$$u_X^{\widehat{\diamond}\mathbb{A}} = \frac{\left(|\mathbb{A}| - \sum\limits_{A\in\mathbb{A}} u_X^A\right) \cdot \prod\limits_{A\in\mathbb{A}} u_X^A}{\left(\sum\limits_{A\in\mathbb{A}} \prod\limits_{A'\neq A} u_X^A\right) - |\mathbb{A}| \cdot \prod\limits_{A\in\mathbb{A}} u_X^A}$$

$$\mathbf{a}_X^{\widehat{\diamond}\mathbb{A}}(x) = \frac{\sum\limits_{A\in\mathbb{A}} \mathbf{a}_X^A(x)\left(1 - u_X^A\right)}{|\mathbb{A}| - \sum\limits_{A\in\mathbb{A}} u_X^A}$$